Computational complexity of Eq. (2) is exponential in the number of neighbouring variables \( |\mathcal{B}| \). A generic dynamic programming solution:

- Assume binary variables \( x_i \in \{0, 1\} \) at \( \mathcal{B} \).

Finish function \( f_K : X_K \rightarrow \mathbb{R} \) can be minimized in time \( O(\alpha) \) with any subset \( K \subset \mathcal{B} \) of its variables fixed.

We can perform Eq. (2) efficiently in \( K = \{x_i\} \).

Idea: break the problem in half, at each iteration. For each half, we minimize

\[
\sum_{x_{i_1}} \ldots \sum_{x_{i_l}} f(x_{i_1}, \ldots, x_{i_l}) = \sum_{x_{i_1}} \ldots \sum_{x_{i_{l/2}}} f(x_{i_1}, \ldots, x_{i_{l/2}} + \ldots + f(x_{i_1}, \ldots, x_{i_{l/2}})
\]

where we need to minimize \( x_{i_1}, \ldots, x_{i_{l/2}} \) and \( x_{i_{l/2} + 1}, \ldots, x_{i_l} \).

Complexity: \( O(\sqrt{K} \log K) \) sorting + cost of minimizing \( f \) in Eq. (5) \( \rightarrow O(\mathcal{K}(\log K)) \).